University of Groningen

The Non Uniqueness of Inverse Problems and Partial Coherence

Hoenders, B.J.; Baltes, H.P.

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THE NON UNIQUENESS OF INVERSE PROBLEMS AND PARTIAL COHERENCE.

B.J. Hoenders, Technical Physical Laboratories, State University at Groningen, Nijenborgh 18, 9747 AG Groningen, The Netherlands.

H.P. Baltes, L.G.Z. Landis & Gyr Zug AG,
Zentrale Forschung und Entwicklung,
CH-6301 Zug, Switserland.

Introduction.

The reconstruction of scatterers or sources from measurements of an observable quantity like the radiation pattern can be divided into three separate problems:

(i) The phase of the field on an arbitrary surface, which quantity is indispensable fro the reconstruction of an object, is to be determined from a measurable quantity, like the radiation pattern or differential cross section. (Phase problem).

(ii) From the knowledge of the field on a surface, the field on the surface of the scatterer or source is to be reconstructed. (Inverse diffraction problem).

(iii) The object is to be reconstructed from the knowledge of the field outside the object. (Inverse source or scattering problem).

(i) The phase problem has a longstanding history in physics and arises in connection with e.g. X-ray diffraction, the theory of partial coherence and potential scattering theory. The retrieval of the phase of a field from a measurable quantity may be achieved using a priori information, like the finite bandwidth of a function, and/or combination of independent measurements like e.g. intensity distributions in a microscope, generated by different settings of the defocusing. A review of the present state of the art, particular of the phase retrieval problem in light- and electron optics has been written by Ferwerda [1].

(ii) The inverse diffraction problem has been considered for the propagation in free space of deterministic vectorial- or scalar fields for special geometries like the plane or the sphere. (For references see Hoenders [2]).

It can be shown that for this case a unique determination of the field, up to the scatterer is always possible given the far field pattern or the values of the field on a plane or a sphere. The validity of these results for arbitrary geometries has been shown recently. Hoenders [2].

Similar results can be obtained in the case of stochastic fields: The first order far field correlations uniquely determine the first order correlations of a scalar- or vectorial field on an arbitrary surface, free space propagation is

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considered.

The inverse diffraction problem has been considered for the case that the field is not propagating in free space, but in a medium, or in the case of wave packets, when an electromagnetic field is present. However, introducing a suitable parametrix representation of the associated Green's functions (tensors), describing the propagation of the fields, similar results as for the case of free space propagation can be derived.

Intimately connected with the inverse diffraction problem is the stability of the reconstruction procedure and the concept of the numbers of degrees of freedom of a field, which may be defined as the number of statistically independent parameters which can be reliably determined.

(iii) The unique determination of an object from a measurable quantity is in general, even in the deterministic case, not possible. Very simple examples are a potential well, which, with a suitable choice of the various parameters will transmit an incoming plane wave without changing its form, up to a constant phase factor, and a slab characterized by a real index of refraction, which, for suitable choices of its thickness, will transmit an incoming plane electromagnetic wave without changing its form, up to a constant phase factor.

Less trivial is the following famous example of Schott of a nonradiating charge distribution: Consider a uniformly charged spherical shell of radius a. The shell will not radiate if its centre is set into motion with period T, provided a is an integral of $\frac{1}{2}cT$, while the orbit need not be circular or even planar. The general theory shows that nonradiating current distributions neither need to have symmetrie properties, nor is its required that e.g. its higher order multipole momonts vanish. The vanishing of the radiation field is solely due to a complicated interference phenomenum.

A general theory of deterministic, nonradiating current distributions has been constructed. We will generalize this theory to first order stochastic current correlations.

We will concentrate on the existence of so-called nonradiating stochastic current distributions and derive a necessary and sufficient condition for a stochastic distribution to be nonradiating. The necessary and sufficient condition for nonradiating deterministic harmonically time dependent current distributions is

$$\underline{\tilde{j}}^{T}(k\underline{s})=0, \tag{1}$$

valid for all values of the unit vector $\underline{s}$, and where $\underline{\tilde{j}}^{T}$ denotes the transverse part $\underline{s} \times \underline{s} \times \underline{\tilde{j}}(k\underline{s})$ of the Fourier transform of the current distribution $\underline{j}(\underline{r})$.

This condition is generalized for first order current correlations to

$$\langle \underline{\tilde{j}}^{T}(k\underline{s}_1)\underline{\tilde{j}}^{T*}(k\underline{s}_2)\rangle=0, \tag{2}$$

for all $\underline{s}_1$ and $\underline{s}_2$, where the brackets denote an ensemble average: current correlations satisfying (2) lead to vanishing first order field correlations outside the support of the currents, Hoenders, Baltes [3]. However, the first order field correlations in an arbitrary surface always lead to nonvanishing far field first order correlations.

1. Ferwerda, H.A., Ch.2 in Inverse Problems in Optics, editor H.P. Baltes, Springer, 1978.
2. Hoenders, B.J., Ch.3. of ref.1.
3. Hoenders, B.J., Baltes, H.P., On the existence of nonradiating stochastic current distributions. Submitted for publication to J. Phys. A.